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INFLUENCE OF VARIABLE PROPERTIES UPON TRANSIENT AND STEADY-STATE FREE CONVECTION

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NOMENCLATURE

a, thermal diffusivity;
b, constant defined by equation (11);
C_p, specific heat;
g, acceleration due to gravity;
Gr_x, Grashof number, $\varepsilon g x^3/v^2$;
Gr_{x_∞}, Grashof number, $\varepsilon g x^3/v_\infty^2$;
Nu_x, Nusselt number, $\alpha x/\lambda$;
Nu_{x_∞}, Nusselt number, $\alpha x/\lambda_\infty$;
Pr, Prandtl number, v/a;
T, absolute temperature;
t, time;
u, velocity in *x-direction;*
u_1, velocity defined by equation (10);
u_{max}, maximum velocity in x-direction; at x = constant plane;
v, velocity in y-direction;
x, distance from leading edge along a plate;
y, normal distance from a plate;
Tr, reference temperature.

Greek symbols
α, heat-transfer coefficient;
δ, boundary-layer thickness;
δ^*, $\int_0^\delta (\rho/\rho_\infty)\,dy$;
ε, $(T_w - T_\infty)/T_\infty$;
η, $\frac{1}{\delta^*}\int_0^y (\rho/\rho_\infty)\,dy$;
θ, dimensionless temperature, $(T-T_\infty)/(T_w-T_\infty)$;
λ, thermal conductivity;
v, dynamic viscosity;
ρ, density;
ω, constant.

Subscripts
w, wall surface;
∞, ambient condition.

1. INTRODUCTION

THE TRANSIENT free convection from a vertical flat plate which is suddenly raised to a uniform higher temperature was analysed by many investigators, for the reason that it is the fundamental problem of unsteady free convection.

But it seems that there is no analysis on unsteady free convection taking the temperature-dependent property fluid into account. In this report, the integral method which was used by Siegel [1] for constant property case has been extended to the form including the variable property effects.

2. BASIC EQUATIONS AND SOLUTIONS

The boundary layer form of the conservation equations which govern the unsteady laminar free convection on a vertical flat plate is written below.

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0, \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = g(\rho_\infty - \rho) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{2}$$

$$\rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right). \tag{3}$$

The equations which express properties of the temperature-dependent property fluid such as air are approximately

$$\frac{\rho}{\rho_\infty} = \frac{T_\infty}{T}, \tag{4}$$

$$\frac{\mu}{\mu_\infty} = \frac{\lambda}{\lambda_\infty} = \left(\frac{T}{T_\infty}\right)^\omega, \tag{5}$$

$$C_p = \text{const.} \tag{6}$$

The velocity and temperature profiles in the boundary layer are assumed satisfying the following boundary conditions in accordance with Karman–Pohlhausen method.

$$y = 0;\quad \theta = 1,\ \frac{\partial}{\partial y}\left(\lambda\frac{\partial \theta}{\partial y}\right) = 0,\ u = 0, \tag{7}$$

$$\begin{aligned} y = \delta;\quad \theta &= \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} = 0,\\ u &= \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} = 0. \end{aligned} \tag{8}$$

Then, each profile is given by

$$\theta = 1 + b\eta - (6+3b)\eta^2 + (8+3b)\eta^3 - (3+b)\eta^4, \tag{9}$$

$$u = u_1\eta(1-\eta)^3, \tag{10}$$

where

$$b = \frac{3}{1-\omega}\cdot\frac{1+\varepsilon}{\varepsilon}\left[-1+\left(1-\tfrac{4}{3}(1-\omega)\frac{\varepsilon}{1+\varepsilon}\right)^{1/2}\right]. \tag{11}$$

The conservation equations (2) and (3) can be integrated and become

$$\frac{1}{20}\cdot\frac{\partial}{\partial t}(u_1\delta_*) + \frac{1}{252}\cdot\frac{\partial}{\partial x}(u_1^2\delta_*) = \frac{\varepsilon}{20}(8+b)g\delta_* - Pra_\infty(\varepsilon+1)^{\omega-1}\frac{u_1}{\delta_*}, \tag{12}$$

$$\frac{1}{20}(8+b)\frac{\partial \delta_*}{\partial t} + \left(\frac{5}{168} + \frac{b}{252}\right)\frac{\partial}{\partial x}(u_1\delta_*) = -\frac{ba_\infty(\varepsilon+1)^{\omega-1}}{\delta_*}. \tag{13}$$

These equations are hyperbolic in type and hence a solution can be found by utilizing the method of characteristics.

Letting I_+ and I_- designate the two families of characteristics, we have

$$\frac{dx}{dt} = \frac{5}{63}u_1\left(1 \pm \frac{1}{(16+2b)^{1/2}}\right); \; I_\pm. \tag{14}$$

Equations (12) and (13) can be solved without difficulty in the pure conduction region at sufficiently short period and in the steady state region at sufficiently long period after the sudden variation of the wall temperature.

The limits of these regions on the $x-t$ plane are determined by integrating equation (14) by use of u_1 in each corresponding simple analytical solution.

1. *The solutions in the initial one-dimensional transient flow development*

For sufficiently short period, the time dependent solutions of the one-dimensional case of equations (12) and (13) are given by

$$\delta_* = 2\left(\frac{-10b}{8+b}(1+\varepsilon)^{\omega-1}\right)^{1/2}(a_\infty t)^{1/2}, \tag{15}$$

$$u_1 = \frac{-2b(8+b)}{-3b+Pr(8+b)}g\varepsilon t. \tag{16}$$

The time at which purely one-dimensional heat conduction is terminated for each position along the plate is determined by the integration of I_+ in equation (14) with resultant u_1 in equation (16), and given by

$$t\left(\frac{g\varepsilon}{x}\right)^{1/2} = \left(\frac{63}{5}\right)^{1/2}\left(\frac{3b-Pr(8+b)}{b(8+b+\sqrt{4+0.5b})}\right)^{1/2}. \tag{17}$$

2. *The solutions in the steady state at sufficiently long period after the sudden variation of wall temperature*

The solution is dependent on x only in this region and given by

$$\delta_* = \left\{\frac{4480(\varepsilon+1)^{2\omega-2}}{Pr^2}\cdot\frac{-b[-10b+3Pr(15+2b)]}{(15-2b)^2(8+b)}\right\}^{1/4} \times Gr_{x\infty}^{-1/4}\cdot x, \tag{18}$$

$$u_1 = \left(\frac{504}{5}\right)^{1/2}\left(\frac{-b(8+b)\varepsilon gx}{-10b+3Pr(15+2b)}\right)^{1/2}. \tag{19}$$

The time which is required to reach steady state is found by integrating I_- in equation (14) with resultant u_1 in equation (19) and then written

$$t\left(\frac{q\varepsilon}{x}\right)^{1/2} = \left(\frac{63}{10}\right)^{1/2}\left(\frac{-10b+3Pr(15+2b)}{-b(8+b)}\right)^{1/2} \times\left[1-\left(\frac{1}{16+2b}\right)^{1/2}\right]^{-1}. \tag{20}$$

3. *Characteristics of initial one-dimensional pure conduction regime*

3.1. In the local Nusselt number:

$$\frac{Nu_{x\infty}}{Gr_{x\infty}^{1/4}} = \frac{(1+\varepsilon)^{\frac{\omega-1}{2}}}{2\sqrt{10}}(-b(8+b)Pr)^{1/2}\left[t\left(\frac{\varepsilon q}{x}\right)^{1/2}\right]^{-1/2}, \tag{21}$$

we have for constant property,

$$\frac{Nu_x}{Gr_x^{1/4}} = \left(\frac{3Pr}{10}\right)^{1/2}\left[t\left(\frac{\varepsilon g}{x}\right)^{1/2}\right]^{-1/2}, \text{ as } \varepsilon \to 0. \tag{22}$$

When $Pr = 1$, the difference of heat-transfer coefficient obtained from equation (22) and Schetz's [3] exact solution is 3%.

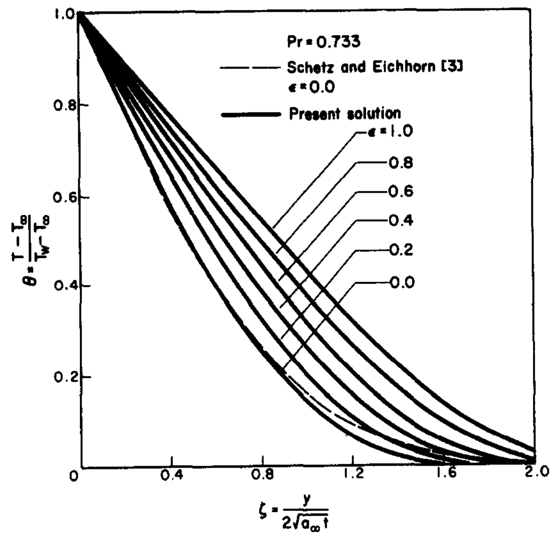

FIG. 1. The effects of variable properties on dimensionless temperature profiles during initial one-dimensional transient flow development.

3.2. In the maximum velocity

$$\frac{u_{max}}{(\varepsilon gx)^{1/2}} = \frac{27}{128}\cdot\frac{-b(8+b)}{-3b+Pr(8+b)}t\left(\frac{\varepsilon g}{x}\right)^{1/2}, \tag{23}$$

we have for constant property,

$$\frac{u_{max}}{(\varepsilon gx)^{1/2}} = \frac{27}{64}\cdot\frac{1}{1+Pr}t\left(\frac{\varepsilon g}{x}\right)^{1/2}, \text{ as } \varepsilon \to 0. \tag{24}$$

When $Pr = 1$, maximum velocity derived from equation (24) is greater than Schetz's [3] exact solution about 4%.

4. *Characteristics of steady state*

4.1 In the local Nusselt number;

$$\frac{Nu_{x\infty}}{Gr_{x\infty}^{1/4}} = -b\left\{\frac{Pr^2(1+\varepsilon)^{2\omega-2}}{4480} \times\frac{(15+2b)^2(8+b)}{-b[-10b+3Pr(15+2b)]}\right\}^{1/4}, \tag{25}$$

we have for constant property,

$$\frac{Nu_x}{Gr_x^{1/4}} = 2\left[\frac{363Pr^2}{4480(20+33Pr)}\right]^{1/4}, \text{ as } \varepsilon \to 0. \tag{26}$$

When $Pr = 0.733$, the difference of R.H.S. of equation (26) and the corresponding value of Ostrach's [4] exact solution is less than 1.5%.

4.2. In the maximum velocity;

$$\frac{u_{max}}{(\varepsilon gx)^{1/2}} = \frac{81}{128}\cdot\left(\frac{-14b(8+b)}{5[-10b+3Pr(15+2b)]}\right)^{1/2}, \tag{27}$$

we have for constant property,

$$\frac{u_{max}}{(\varepsilon gx)^{1/2}} = \frac{81}{64}\left(\frac{42}{5(20+33Pr)}\right)^{1/2}, \text{ as } \varepsilon \to 0. \tag{28}$$

When $Pr = 0.733$, the difference of maximum velocity obtained from equation (28) and Ostrach's [4] exact solution is about 1.8%.

5. *Reference temperature*

The temperature to which fluid properties are referred in constant property formula (22) may be determined by giving the minimum difference of heat-transfer coefficient derived from equation (22) and variable property formula (21). Then, this reference temperature is given by

$$Tr = T_\infty + 0.78(T_w - T_\infty), \tag{29}$$

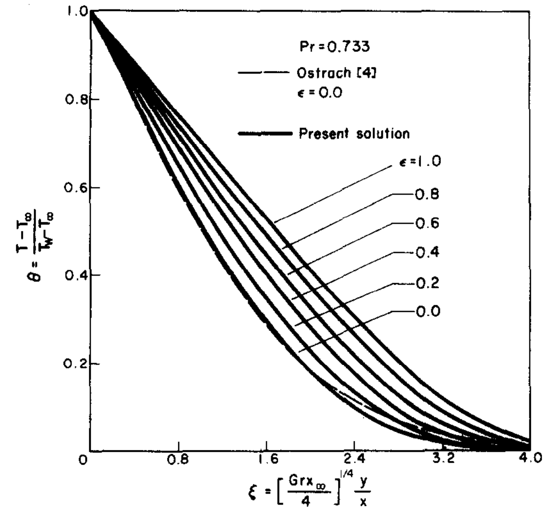

FIG. 2. The effects of variable properties on dimensionless temperature profiles for steady state.

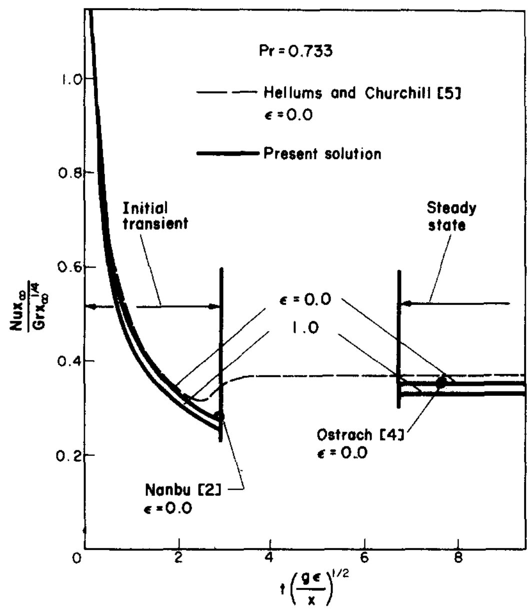

FIG. 3. The effects of variable fluid properties on transient Nusselt numbers.

for $Pr = 0.733, \omega = 0.76$. If the fluid properties referred to the temperature given by equation (29) are used in equation (22), the difference of heat-transfer coefficient derived from (21) and (22) is less than 0.1% for ε which is in between -0.55 and 3. Similarly, for steady state heat-transfer coefficient derived from equation (26), the reference temperature is given by

$$\begin{gathered} Tr = T_\infty + 0.73\,(T_w - T_\infty), \\ \text{for } -0.3 < \varepsilon < 3, \\ Pr = 0.70,\ \omega = 0.75. \end{gathered} \tag{30}$$

3. NUMERICAL RESULTS

Numerical results have been obtained for air ($Pr = 0.733$, $\omega = 0.76$). The temperature profiles for various ε in the initial transient and steady state are given in Fig. 1 and Fig. 2. In these figures the results of the more exact constant property solutions are expressed by broken lines for the comparison. The effects of the variable fluid properties on the transient and steady-state Nusselt numbers are illustrated in Fig. 3.

The limits of the regions for initial pure conduction and steady state taking place are also shown in Fig. 3, but the effect of variable fluid properties on these limits is too small to distinguish in this figure. Local Nusselt number at end of pure conduction together with its time for constant property case taken from Nanbu's more exact solution [2] is shown in Fig. 3 for the comparison. The ratio of steady state heat-transfer coefficient obtained from constant property formula (26) and variable property formula (25) for various reference temperature in the constant property formula is shown in Fig. 4 together with the results from Sparrow's more exact solution [7] expressed by broken lines. It seems that the results of this approximate method are more accurate for steady state than initial transient free convection. But it might be inferred by Goodman's comparison [8] that present method gives the reasonable approximation also for the variable fluid property effects on initial transient free convection.

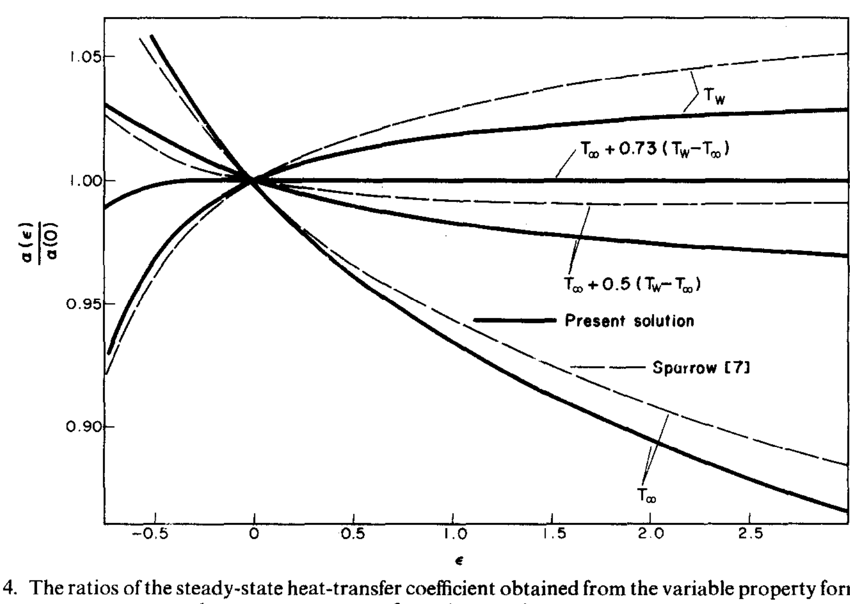

FIG. 4. The ratios of the steady-state heat-transfer coefficient obtained from the variable property formula to the constant property formula at various temperature.

4. CONCLUSION

The following might be concluded:

1. Present results give the reasonable approximation for the variable fluid property effects on the initial transient and steady-state free convection.

2. The simple analytical expressions of the free convection characteristics have been obtained.

3. The time required for purely one-dimensional heat conduction being terminated and the time required to reach steady-state are hardly influenced by the effects of the variable fluid properties.

4. Present method might be able to be improved its accuracy and adaptability by using more accurate form of the profiles and expressions of variable properties.

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STUDY OF ACOUSTIC PHENOMENA ACCOMPANYING NUCLEATE BOILING OF SUBCOOLED DILUTE AQUEOUS ETHANOL SOLUTIONS AND AQUEOUS SURFACTANT SYSTEMS

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INTRODUCTION

A VAST amount of literature has been published describing both the experiments involving nucleate boiling of pure liquids and the elucidation of the heat-transfer mechanism for this boiling process from theory. Relatively little data have been collected for binary liquids and the dominant theoretical contributions in this area have been made by Van Stralen and his coworkers [1]. The study of binary systems is of potential industrial importance since a number of workers have demonstrated that the introduction of a small amount of a second component such as ethanol, l-butanol etc. to water results in an increase in nucleate boiling heat transfer by as much as 400% [2–4]. Very recently Ponter *et al.* [5] showed for surface tension positive systems that the heat flux vs concentration relationship exhibited a strong maximum which coincided with a maximum in the contact angle measured under boiling conditions and obtained when the vapor and surrounding liquid were of the same composition and not in equilibrium. To obtain more information on the state of the bubble forming under boiling conditions an experimental program was undertaken to examine the sound emissions using a technique similar to that described by Ponter and Haigh previously [6].

EXPERIMENTAL

The liquid systems examined were water, aqueous ethanol mixtures, aqueous lauryl alcohol mixtures and dilute sodium lauryl sulphate solutions. The simple pool boiling apparatus was constructed in which boiling was induced on the surface of a platinum wire immersed in a quiescent pool of water by the passage of an electric current (DC). Details of the electric circuit are similar to those described previously by Haigh and Ponter [7]. The boiling sounds were detected by a Brüel and Kjaer hydrophone—type 8100—containing a lead zirconate pressure transducer having a flat frequency response characteristic over a large frequency range. The hydrophone was immersed in the water and positioned at the same depth as the wire. The signal from the hydrophone was fed to a Brüel and Kjaer—type 2120—frequency analyzer which scanned automatically from 2 to 20 KHz. The analyzer was coupled to a Brüel and Kjaer—type 2305—logarithmic recorder by a flexible drive from the recorder motor so that coincidence between the analyser frequency and the recorder chart calibration resulted. The measuring amplifier—type 2626—were used to simultaneously record the intensity of sound against the frequency on precalibrated paper. Data were collected for a fixed constant bulk liquid subcooling of 6°C, this being achieved by incorporating an additional hot plate into the system which brought the liquid up to the required bulk temperature before measurements were taken.

DISCUSSION

Firstly it was noted for all of the systems examined that changes in liquid concentration affected the intensity of sound emitted by the system but that the frequency spectra was not influenced.

For the ethanol–water mixtures an increase in concentration from zero to 10 mol % resulted in a steady decrease in the sound intensity. As described previously, both the